Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-09-01

Conceptual questions

Question 1. Determine which of the following polar coordinates (r, θ) does NOT represent the same point as the other four:

$$(3, -5\pi/4), (3, 3\pi/4), (-3, -\pi/4), (-3, 3\pi/4), (-3, 7\pi/4)$$

Question 2. What shape in the xy-plane does the polar curve $r = \csc\theta$ describe?

Question 3. One the backside of this sheet of paper, indicate **all** regions in the given r, θ-grid that correspond to the shaded regions A and B in Figure 1.

Computations

Problem 1. Find the slope of the tangent line to the polar curve $r = 1/\theta$ at the point where $\theta = \pi$.

Problem 2. Find a polar equation $r = f(\theta)$ for the circle centered at the point (a, b) (given in Cartesian coordinates) passing through the origin[1]. What θ interval traces out your circle once?

Problem 3. Consider the portion of the spiral $r = \theta$ with $2\pi/3 \le \theta \le 5\pi/6$. See Figure 2. Compute the area *underneath* this curve in two ways:

(a) Convert to parametric equations and use methods of §10.2.
(b) First compute the area of the region with corners O, B, and D using methods of §10.4. Then use that to find the desired area. **Hint:** Think about the right triangles $\triangle$BAO and $\triangle$DCO.

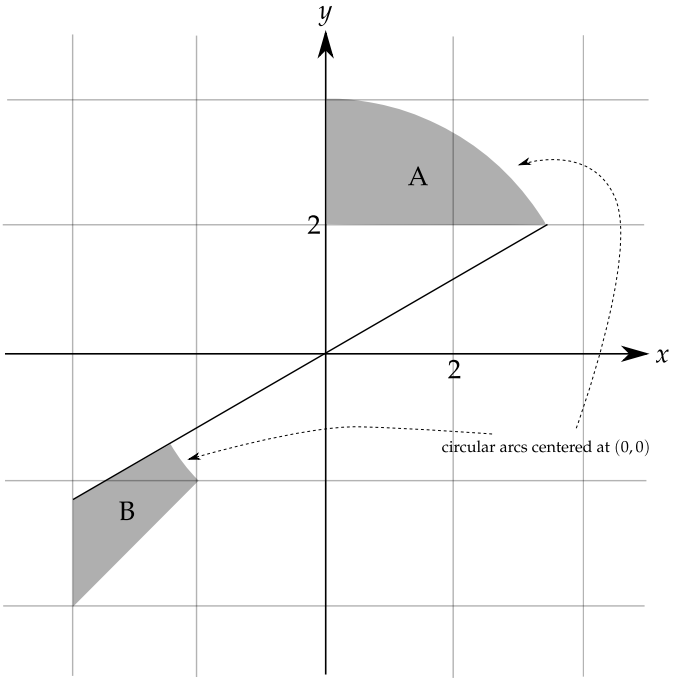


FIGURE 1. The two curved arcs are parts of circles centered at the origin. All other sides are straight lines.

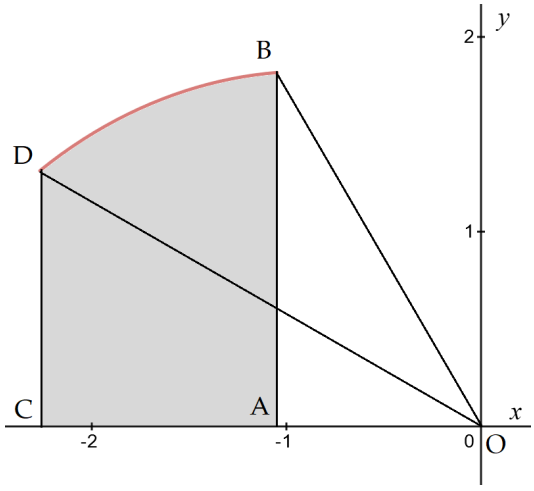
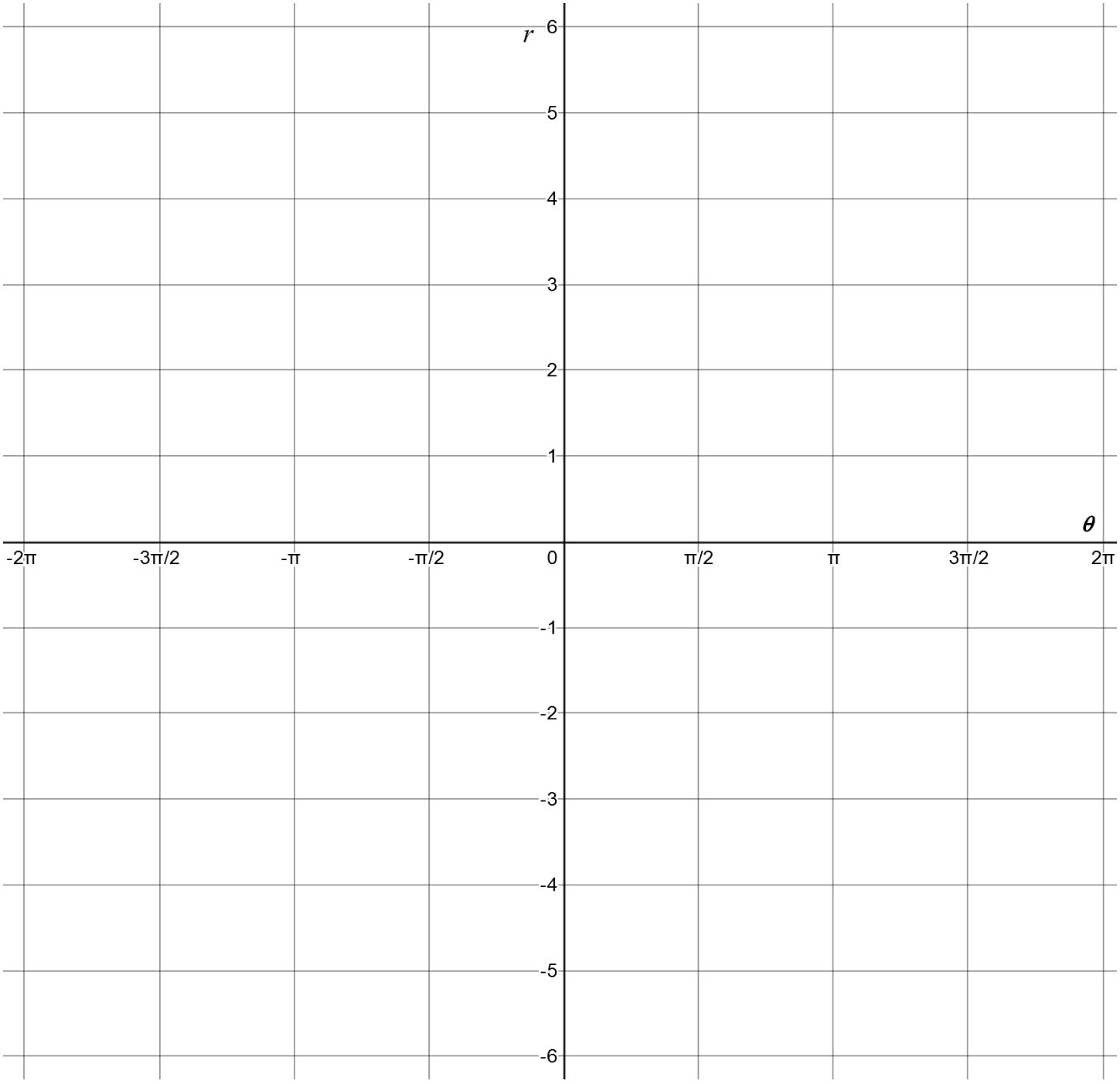


FIGURE 2. The setup of Problem 3.

[1]The only circles which have a "nice" polar form are those centered at the origin or passing through the origin.


r
θ
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-2π
-3π/2
-π
-π/2
0
π/2
π
3π/2
2π